JOURNAL OF AIRCRAFT
Vol. 45, No. 4, July–August 2008

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Rapid Estimation of Airfoil Aerodynamics for Helicopter Rotors

Antonio Filippone*
University of Manchester,
Manchester, England M60 1QD, United Kingdom

DOI: 10.2514/1.35560

I. Introduction

THE calculation of helicopter rotor performance with comprehensive rotorcraft codes relies on good-quality airfoil data. The data must be arranged in matrix form to cover a range of angles of attack, Reynolds numbers, and Mach numbers. Interpolation is done at intermediate values. It turns out that these data are not always available; they may be incomplete or even unreliable. Operation at transonic speeds is known to be problematic. Additional phenomena (unsteadiness, turbulence effects, etc.) take place that make the data relatively scattered. Computational methods have progressed in the low-speed area. At present, it is possible to accurately predict the aerodynamics at moderately high Reynolds numbers and low Mach numbers. These methods can be reasonably applied to generate airfoil polars up to the static stall. Beyond this point, the calculation of the aerodynamic coefficients is often inaccurate and requires careful validation against verified wind-tunnel experiments. The problem addressed by this study is the development of semi-empirical calculation methods to produce airfoil charts over a range of Reynolds and Mach numbers. The methods proposed do not require direct aerodynamic calculations beyond the low Mach number polar. The problem of the Reynolds and Mach number effects are discussed for the lift-curve slope C_{L_α}, the lift coefficient C_L, the maximum-lift coefficient $C_{L_{\max}}$, and the drag coefficient C_D.

To validate and verify the theory, one must rely on wind-tunnel data. It turns out that the wind-tunnel data suffer from some uncertainty of their own. The uncertainty depends on a variety of parameters, including the wind tunnel itself, the accuracy of the balance, the data recording system, blockage corrections, freestream turbulence, the fidelity of the airfoil model, the surface finish, and other factors. Some aspects of wind-tunnel testing, including a critical assessment of the data, are available in McCroskey [1] (for the NACA 0012 airfoil) and Bousman [2] (for the SC1095 and SC1094 airfoils). These data have been previously assessed by Totah [3] on the basis of eight different experimental campaigns. The NACA 0012 is the most studied airfoil. Its application to rotorcraft is historically important, because all helicopter rotors before 1965 used this airfoil alone. The airfoil has found widespread use to the present time.

Both McCroskey and Bousman investigated the accuracy of about a dozen experimental data sets and extrapolated some correlation curves. They provided conclusions regarding the validity of each data set. In brief, all the experiments available present aspects of considerable inconsistency. McCroskey concluded that only a few tests (indicated as group 1) were trustworthy in terms of the accuracy of the C_{L_α} and C_{D_o}. Even then, these tests were limited in the range of angle of attack and Mach number.

Dadone [4] compiled data sets for over a dozen rotorcraft airfoils and produced charts of aerodynamic coefficients based on various collections of wind-tunnel tests. In this report, there is no specific discussion regarding the accuracy of the data.

Semi-empirical theories do exist. For example, Yamauchi and Johnson [5] analyzed the effects of Reynolds number for a large number of rotorcraft airfoils, in particular the effects of Reynolds number on the minimum drag and maximum-lift coefficients. The method proposed by Beddoes [6] is powerful for extrapolating the airfoil performance over a range of Mach numbers and angles of attack if the separation point on the suction side of the airfoil is known. The method relies on the Helmholtz or Kirchhoff solution for the lift equation past a flat plate. However, the separation point is generally not known, and there is no other way to get this datum without wind-tunnel testing.

At the other end of the computational spectrum there is computational fluid dynamics (CFD). Smith et al. [7] performed detailed CFD calculations on the SC1095 based largely on the results presented by Bousman. These authors used five different CFD codes and concluded that the numerical results are "as good as" the experimental data, although there is a relatively large scatter between the computed data.

II. Aerodynamic Models

The lift coefficient is expressed as

$$C_L = C_{L_o}(M) + C_{L_\alpha}(\alpha, M)\alpha \tag{1}$$

The zero-lift angle of attack is only weakly dependent on the Mach number. Alternatively, if $*$ denotes reference (known) data, we can assume

$$C_{L_o} = C_{L_o}(M^*) + (\Delta C_{L_o}/\Delta M)(M - M^*) \tag{2}$$

The lift-curve slope depends on both the angle of attack and the Mach number. A suitable correction for the lift-curve slope is done by using the Kármán–Tzien equation:

$$C_{L_\alpha}(\alpha, M) = \frac{C_{L_\alpha}(\alpha, M^*)}{\beta}$$
$$\beta = \sqrt{1 - M^2} + \frac{1}{2}\frac{M^2}{1 + \sqrt{1 - M^2}} \tag{3}$$

This correction is valid at low subsonic speeds. The lift-curve slope drops dramatically in the transonic region, although the lift-curve slope increases with the Mach number. As a consequence, if the angle of attack is fixed, then the C_L produced at transonic Mach numbers is higher than the value at the reference Mach, M^*. If $M > M_{dd}$, the procedure will have to be modified. From the analysis

Received 8 November 2007; revision received 27 February 2008; accepted for publication 28 February 2008.

*School of Mechanical, Aerospace, and Civil Engineering, George Begg Building, Post Office Box 88; a.filippone@manchester.ac.uk. Senior Member AIAA.

of several experiments, McCroskey [1] concluded that the best curve fit for the lift-curve slope of the NACA 0012 is

$$\beta_1 C_{L_\alpha} = 0.1025 + 0.00485 \log\left(\frac{Re}{10^6}\right) \tag{4}$$

with a maximum error of 0.0029. Equation (4) uses the Prandtl–Glauert compressibility correction, rather than the Kármán–Tzien. The drag coefficient can be expressed as

$$C_D(M) = C_D(\alpha, M^*) + \Delta C_D(M) \tag{5}$$

Two corrections to Eq. (5) are required. First, we need a Reynolds number effect, due to the increased freestream due to an increased Mach number. Second, we need a Mach number correction to operate around the drag divergence point and beyond. Assume that the airfoil polar is calculated at the reference Mach number M^*. The Reynolds number effect is calculated from the definition of the Mach number, $Re = Ul/\nu = Mac/\nu$, where c is the chord and a is the speed of sound. If the atmospheric conditions are fixed, then

$$Re(M) = Re^* + Re^*(M - M^*) \tag{6}$$

Various semi-empirical expressions exist to correlate the local skin friction and drag coefficients to the Reynolds number. In this context, we assume that the profile drag scales with the Reynolds number according to Prandtl–Schlichting:

$$C_D \propto \frac{1}{(\log_{10} Re)^{2.548}} \tag{7}$$

Hence, the Mach number effect on the profile drag coefficient becomes

$$\frac{C_D}{C_D^*} = \frac{\log_{10} Re^*}{\log_{10} Re} \tag{8}$$

Equation (8) is not dependent on any additional parameters. The next step is to calculate the divergence Mach number of the airfoil. This is done through a modified version of Korn's equation:

$$M_{dd} = \kappa_A - \kappa C_L - (t/c) \tag{9}$$

with κ_A variable from 0.88 (NACA airfoils) to about 0.95 (modern supercritical wing sections); κ is a lift-induced factor. This method was originally developed in the 1970s for fixed-wing aircraft and subsequently applied by Mason [8] and Malone and Mason [9]. In the present formulation, the C_L to be used in Eq. (9) must be the *corrected* value, obtained by a combination of Eqs. (1) and (3) (recall that the effective angle of attack is fixed). The effect of the airfoil thickness is approximate, but in line with some experiments published by Harris [10]. Although Harris' results could not be related to thickness only, they showed that, for a thickness of 10–11%, the change in divergence Mach number was $\Delta M_{dd} \simeq t/c$ for a given normal force coefficient.

For nonlifting conditions, the M_{dd} is proportional to κ_A. If the M_{dd} can be inferred from the reference data, then κ_A can be found directly from Eq. (9). For example, at $C_L = 0$ the NACA 23012 has $M_{dd} = 0.791$ (Bingham and Noonan [11]). This yields $\kappa_A \simeq 0.91$. Likewise, for the NACA 0012 we estimate $\kappa_A \simeq 0.88$. At lifting conditions, the movement of the M_{dd} is proportional to the C_L. The change in M_{dd} with the lift coefficient, or angle of attack, is

$$\partial M_{dd}/\partial\alpha = -\kappa C_{L_\alpha}, \qquad \partial M_{dd}/\partial C_L = -\kappa \tag{10}$$

The analysis of various experimental data for the NACA 0012 and 23012 indicate that $\kappa \simeq 0.22$–0.24. In absence of more accurate data, the value $\kappa \simeq 0.2$ can be assumed in Eq. (9).

Bingham [12] concluded that the M_{dd} decreases almost linearly with the increasing C_L. The most notable departures to this linearity are as follows: negative C_L, very large C_L, and airfoils with a large leading-edge radius. Some of these data have been extrapolated. The linearized M_{dd}–C_L relationship is:

$$\Delta M_{dd} \simeq -0.214\Delta C_L, \qquad \Delta M_{dd} \simeq -0.227\Delta C_L \tag{11}$$

for the NACA 0012-33 and 23012-33, respectively. There is a relationship between the wave drag and the critical Mach number. This relationship was proposed by Lock (1940s):

$$\Delta C_D = 20(M - M_c)^4, \qquad M > M_c \tag{12}$$

A relationship between M_c and M_{dd} can be calculated from Eqs. (9) and (12). In fact, derive Eq. (12) and recall the definition of M_{dd}:

$$\left(\frac{\mathrm{d}C_D}{\mathrm{d}M}\right)_{Mdd} = 0.10 = 80(M_{dd} - M_c)^3 \tag{13}$$

The corresponding drag coefficient is now calculated from Eq. (12). A problem arises in the choice of the C_L. In fact, as the Mach number rises, the C_L calculated from Eq. (1) tends to increase, due to an increase in the lift-curve slope. Therefore, the problem is nonlinear and must be solved iteratively.

Along with the airfoil polar, the data required for the extrapolation include the relative thickness, t/c; the reference Mach number, M^*; and the reference Reynolds number, Re^*. The Korn factor κ_A is a free parameter that must be chosen carefully, because the transonic effects are strongly dependent on it. This problem can be overcome if the M_{dd} is known at one value of the C_L.

It is possible to verify that the choice of the Korn factor is correct. First, we need to find a best fit of the wind-tunnel data; then, we need to calculate the M_{dd} according to the first equivalence in Eq. (13); and finally, we need to calculate κ_A by solving Eq. (9). There is some arbitrariness in fitting the reference data, but we find that the best correlation is a polynomial of order 3. This gives $M_{dd} = 0.818$, a figure very close to $M_{dd} = 0.814$ as reported by Bousman [2]. The solution of Eq. (9) yields $\kappa_A \simeq 0.91$. However, note that, due to the scatter of the wind-tunnel data around $M = 0.7$–1.8, a better approximation cannot be achieved.

Figure 1 shows the comparison between the present model and the wind-tunnel results of Bingham and Noonan [13]. From the nonlifting conditions, it was found that $\kappa_A \simeq 0.911$. The comparison between the calculation and wind-tunnel data is good at most angles of attack.

The $C_{L_{\max}}$ is known to be dependent on both the Reynolds and the Mach numbers. Generally, an increase in the Reynolds number at a fixed Mach number tends to increase the $C_{L_{\max}}$. Fitting the available data can be done with a variety of functions that are accurate within a certain range of Re or M. In the present approach, the dependence of the $C_{L_{\max}}$ on the Mach number is calculated from

$$C_{L_{\max}}(M) \simeq \log M^b + s \tag{14}$$

The shift parameter s is calculated from the data at the reference conditions (Re^* and M^*):

$$s = C^*_{L_{\max}} - \log M^b_* \tag{15}$$

By inserting s into Eq. (14), we find the correlation between the loss of $C_{L_{\max}}$ and the increase in Mach number:

$$\Delta C_{L_{\max}} \simeq -\log(M/M^*)^b \tag{16}$$

In the absence of detailed data, it is reasonable to assume $b \simeq -1/2$. The effect of the Mach number is shown in Fig. 2. The coefficients of the regression curve, Eq. (14), are $b = -1/2$ and $c = 0.71$. In both cases, the regression curve is within the level of confidence of the wind-tunnel data. As the Mach number increases, the maximum-lift angle $\alpha_{\max}$ decreases. There remains the problem of how to model the airfoil behavior in deep stall. The experimental data indicate that the stall becomes smoother as the Mach number increases and, in some cases, the C_L increases again. The problem cannot be treated without making arbitrary empirical assumptions.

The test case was the rotor of the Sikorsky UH-60 Black Hawk helicopter. The rotor blade has three airfoil sections: SC1095 up to $r/R = 0.48$, SC1094-R8 up to $r/R = 0.84$, and SC1095 at the outer

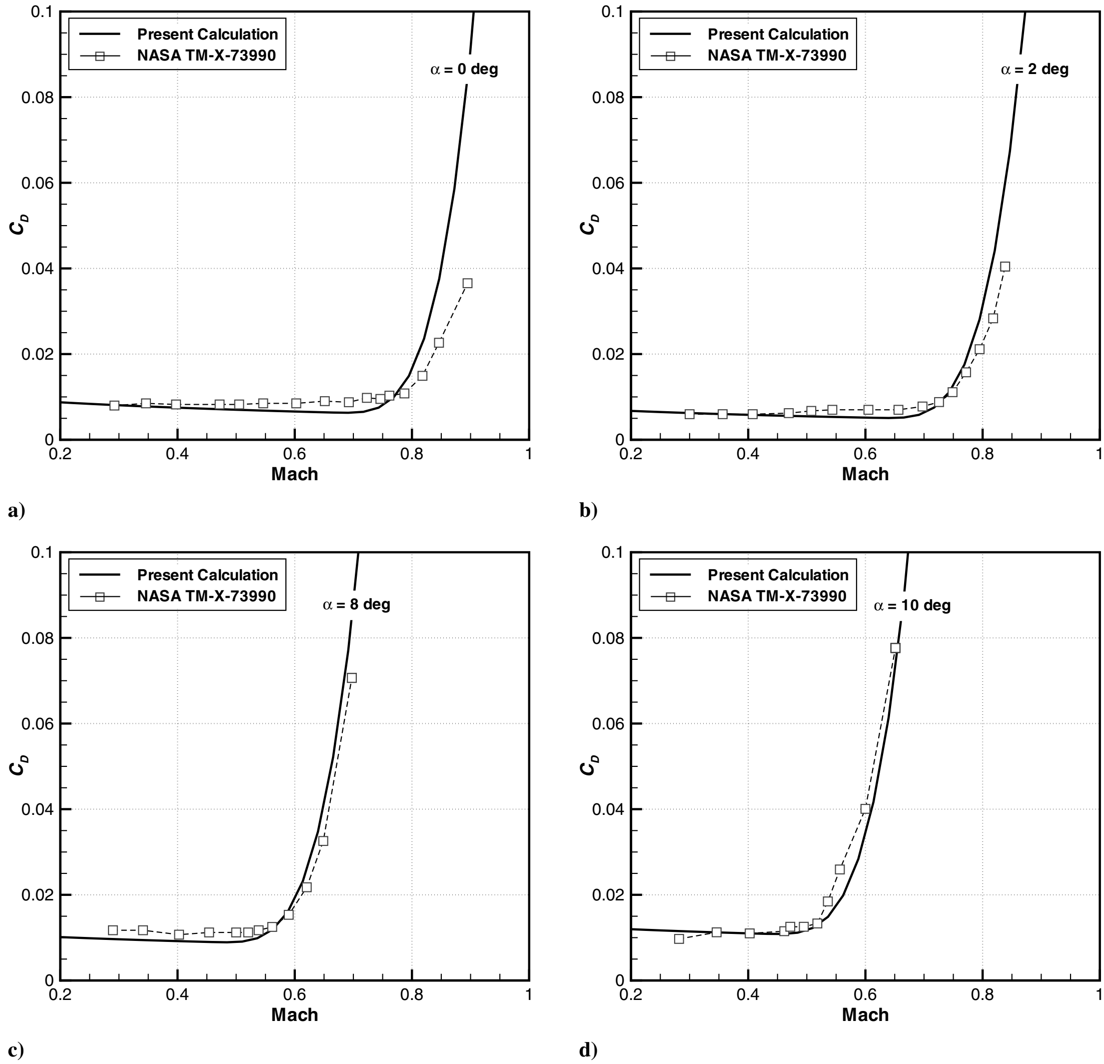

Fig. 1 Drag rise of the NACA 23012 airfoil and a comparison with the experiments of Bingham and Noonan [13].

board. The tip Mach number in this case was $M_{\mathrm{tip}} = 0.650$. The SC1095 data for this calculation are shown in Fig. 3. The data shown are the C_L polar, the drag polar, the transonic drag rise, and the polar for the pitching moment, which is calculated following the same method as the wave drag.

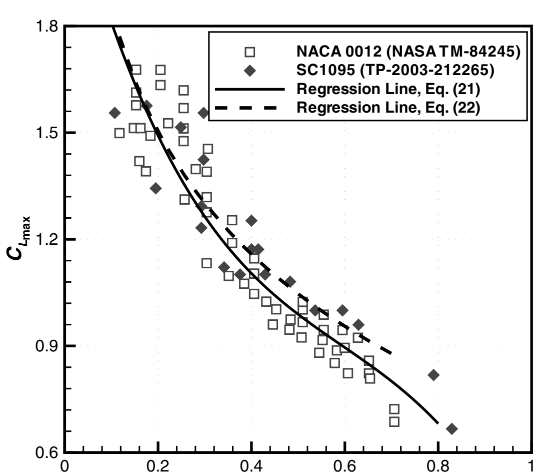

Fig. 2 Estimated $C_{L_{\max}}$ from the wind-tunnel data for the NACA 0012 and SC1095.

III. Conclusions

Semi-empirical methods have been proposed to extrapolate airfoil aerodynamics at high Reynolds numbers and at transonic Mach numbers. The theory has been compared with the available wind-tunnel data for some rotorcraft airfoils: the NACA 0012, the NACA 23012, the SC1095, and the SC1095-R8. Comparison with the data shows that the Reynolds number effects on the C_D and the $C_{L_{\max}}$ can be captured with the same order of accuracy as the wind-tunnel data by using boundary-layer theory.

The accuracy was verified for the NACA 0012, the NACA 23012, and the SC1095. The lift-curve slope can be calculated with engineering accuracy before the lift stall. The accuracy has been verified with the NACA 0012 and the SC1095. An iterative procedure around M_{dd} allows the calculation of the point of transonic dip. However, the latter procedure could not be verified.

The M_{dd} can be calculated accurately at zero angle of attack and, for some cases, at all useful angles of attack. This accuracy was verified for the NACA 0012, the NACA 23012, and the SC1095. The critical parameter is the Korn factor or the M_{dd} at nonlifting conditions. The latter parameter is preferred.

The semi-empirical relationships proposed are valid up to about $M = 0.90$. It cannot accurately predict nonlinear phenomena such as lift stall, Mach tuck on the pitching moment, or the drag recovery at low supersonic speeds. The method proposed allows the generation of tabulated data for airfoil aerodynamics (C_L, C_{L_α}, C_D) over a wide range of angles of attack, Reynolds numbers, and Mach numbers.

The data required include the airfoil thickness, the reference Reynolds and Mach numbers, the M_{dd} at nonlifting conditions, and

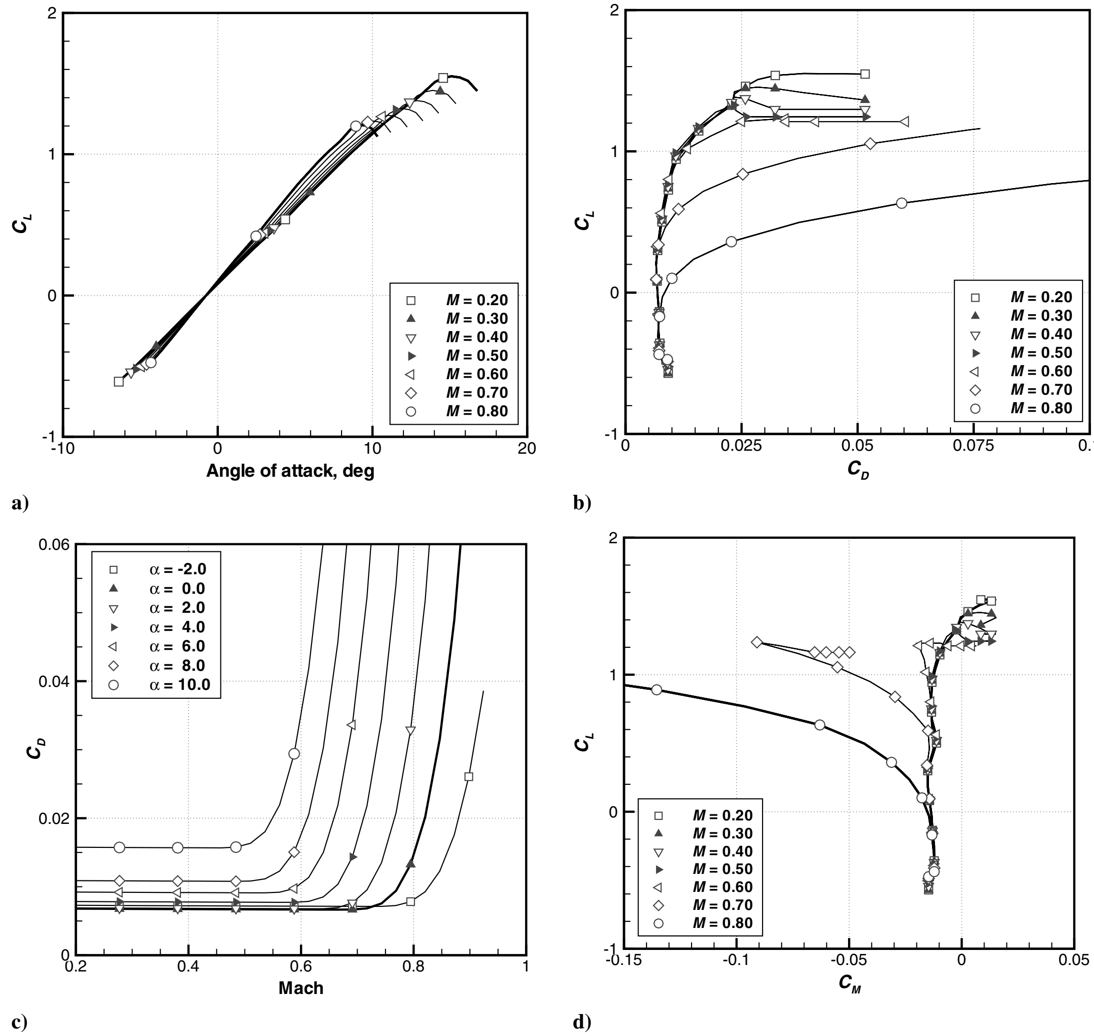

Fig. 3 Aerodynamic polars of the SC1095 (calculated).

the corresponding aerodynamic polar. Like the current CFD analysis, the methods introduced in the present analysis are applicable to steady-state or quasi-steady conditions.

References

[1] McCroskey, J., "A Critical Assessment of Wind Tunnel Results for the NACA 0012 Airfoil," NASA TM-100019, Oct. 1987.

[2] Bousman, W. G., "Aerodynamic Characteristics of SC1095 and SC1094 R8 Airfoils," NASA TP-2003-212265, Dec. 2003.

[3] Totah, J., "A Critical Assessment of UH-60 Main Rotor Blade Airfoil Data," AIAA Paper 93-3413, 1993.

[4] Dadone, L. E., "US Army Helicopter Design Datcom Vol. 1: Airfoils," NASA CR-153247, May 1976.

[5] Yamauchi, G. K., and Johnson, W., "Trends of Reynolds Number Effects on Two-Dimensional Airfoil Characteristics for Helicopter Rotor Analyses," NASA TM-84363, April 1983.

[6] Beddoes, T. S., "Representation of Airfoil Behavior," *Vertica*, Vol. 7, No. 2, 1983, pp. 183–197.

[7] Smith, M. J., Wong, T. C., Postdam, M., Baeder, J., and Phanse, S., "Evaluation of CFD to Determine Two-Dimensional Airfoil Characteristics for Rotorcraft Applications," *60th AHS Forum*, American Helicopter Society, Alexandria, VA, June 2004.

[8] Mason, W. H., "Analytic Models for Technology Integration in Aircraft Design," AIAA Paper 90-3262, Sept. 1990.

[9] Malone, B., and Mason, W. H., "Multidisciplinary Optimization in Aircraft Design Using Analytic Technology Models," *Journal of Aircraft*, Vol. 32, No. 2, March 1995, pp. 431–438.

[10] Harris, C. D., "Aerodynamic Characteristics of Two NASA Supercritical Airfoils with Different Maximum Thickness," NASA TM-X-2532, April 1972.

[11] Bingham, G. J., and Noonan, K. W., "Aerodynamic Characteristics of Several Rotorcraft Airfoils at Mach Numbers from 0.35 to 0.90," NASA TM-X-73990, 1977.

[12] Bingham, G. J., "An Analytical Evaluation of Airfoil Sections for Helicopter Rotor Applications," NASA TN-D-7796, Feb. 1975.

[13] Bingham, G. J., and Noonan, K. W., "Two-Dimensional Aerodynamic Characteristics of Three Rotorcraft Airfoils at Mach Numbers from 0.35 to 0.90," NASA TP-2000, 1982.